Addition and Subtraction

In Addition and Subtraction, we have 5 key teaching concepts:

1. Mental methods before written methods when solving problems.
2. Understanding the relationship between addition and subtraction.
3. Seeing subtraction as finding the difference, not just take-away.
4. Correct terminology: regrouping (addition) and exchanging (subtraction).
5. Strategies used for whole numbers can be applied to decimals in the same way.

1. Mental methods before written methods when solving problems.

When tackling addition and subtraction problems, pupils should always be encouraged to see if they can complete the calculation in their heads or with jottings first before they go straight to a formal written method. It may be quicker and more efficient as formal written methods can be time consuming and do not help develop conceptual understanding.

2. Understanding the relationship between addition and subtraction.

It is important pupils understand that rather than there being 4 operations (+, −, x, ÷), there are 2 relationships. The relationship between addition and subtraction and the relationship between multiplication and division. We want to encourage pupils to use the inverse when solving addition or subtraction calculations mentally.

3. Seeing subtraction as finding the difference, not just take-away.

Often, pupils will only see subtraction as ‘take-away’. This can lead to inefficient methods when subtracting. If pupils understand that ‘subtraction’ means ‘difference’ they can use addition to ‘count on to find the difference’. Many pupils find addition easier than subtraction.

4. **Correct terminology: regrouping (addition) and exchanging (subtraction)**.

The vocabulary we use with pupils when modelling the column method is really important so that it builds on the pupils’ place value understanding.

At Third Space learning, we use ‘exchanging’ when subtracting and ‘regrouping’ when adding. It is helpful, in terms of developing place value and conceptual understanding, to try and remember to maintain the integrity of a number when adding and subtracting numbers.
For instance, when working in the tens column, it is best to talk about subtracting 4 tens from 7 tens, rather than 4 from 7, or if working in the hundreds, adding 3 hundreds to 5 hundreds etc. It can be detrimental if pupils believe they are only ever subtracting or adding ones.

Addition and Subtraction

5. Strategies used for whole numbers can be applied to decimals in the same way.

It is good educational practice to have consistency in methods using whole numbers and decimal numbers. When using terminology we can also use it in the same way for decimals as we do for whole numbers.

We focus on a few key structures:

- Addition: aggregation (combining two amounts) augmentation (increasing an amount which is often linked to counting on a number line).
- Subtraction: take-away, reduction (counting back on number line), inverse of addition (counting on to reach target number), difference.

Number facts

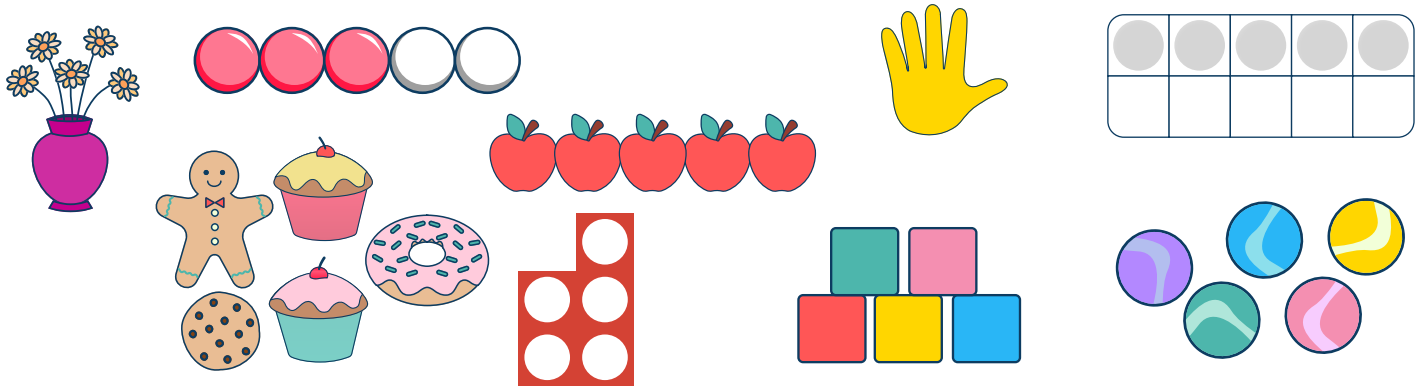
Concept(s)

Number pairs within 10
Pupils learn about number pairs within 10 (for example 2 + 3 = 5).
Introduced in Year 1

Number pairs that make 10
Pupils apply their prior learning to find number pairs that make 10. This involves finding fact families (addition and subtraction calculations).
Introduced in Year 1

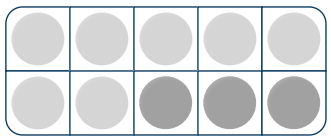
<u>Toys</u>

Initially, introduce number pairs using familiar resources, such as toys / fingers. Pupils should understand different ways to make a number. All the representations are different but they all show the same number.



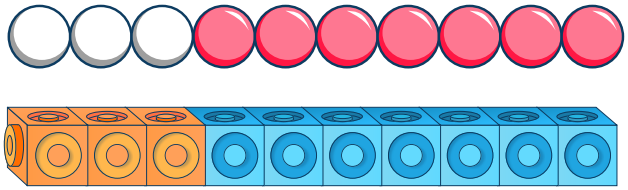
<u>Counters / Tens frame</u>

Counters can be used alone or with a tens frame. Using a tens frame can help pupils identify patterns when making 10. This relates to subitising (the ability to look at a small number of objects and identify how many there are without counting).



<u>Bead string</u>

Multi-link and bead strings can be used to represent different numbers. The colours of the multi-link can represent different parts that the whole can be broken into.



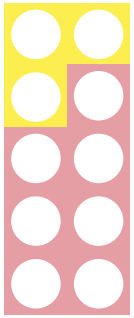
Number facts

Concept(s)

Number pairs within 10
and
Number pairs that make 10
Continued

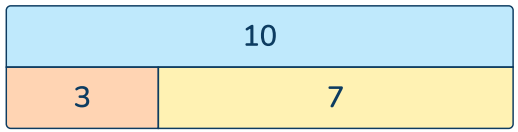
Number shapes

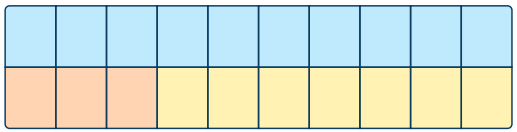
Number shapes can be used in many different ways. When making a number, pupils can place the number shape that they are making first (e.g. if they are making 10, they place 10 first) and put other number shapes on top of this. E.g. in this image, 3 + 7 = 10 or 7 + 3 = 10 as the 3 and 7 number shapes can be placed together to make one 10. It also shows 10 – 3 = 7 or 10 – 7 = 3.



Bar model

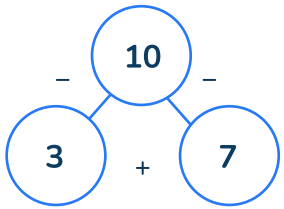
Bar models can be shown as discrete (divided into all the equal parts) or continuous (one bar to represent one number). The bars should (as far as possible) be in proportion.





Part-whole model

When using concrete resources, the language of 'whole' and 'parts' are introduced. Part-whole models can be drawn to help pupils identify the whole and the parts that can make the whole.



Written form

Addend + addend = sum
e.g. 3 + 7 = 10
Minuend – subtrahend = difference
e.g. 10 – 3 = 7

Number facts

Concept(s)

Adding and subtracting by making 10

When crossing a tens boundary through addition or subtraction, pupils can use number pairs to help make the calculation easier.

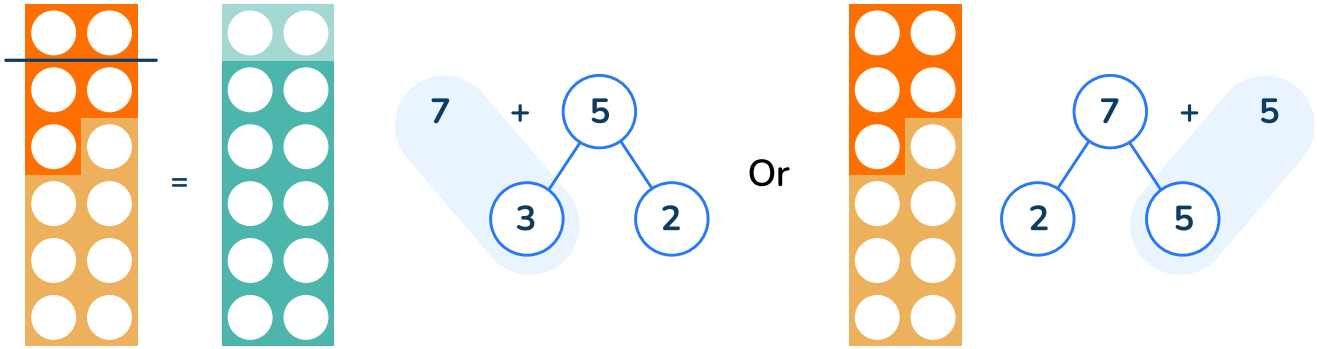
Pupils should use their understanding of number pairs within 10 to help partitioning a number to make a ten.

Introduced in Year 2

Number shapes and part-whole models

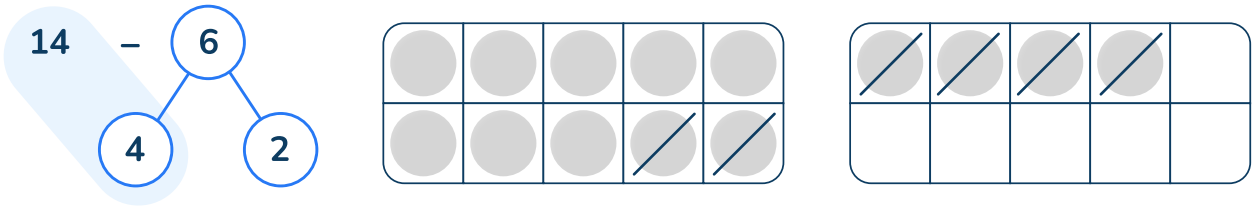
The examples below use number pairs to 5 to make 10. Pupils should identify the number pairs to 5 (0 and 5, 1 and 4, 2 and 3) and identify that 2 and 3 will help them to make 10.

To solve 7 + 5, we can add 3 to 7 then add 2 or we can add 5 to 5 then add 2.



Tens frames and part-whole models

To solve 14 – 6, we can take away 4 from 14, and then subtract 2.



Number facts

Concept(s)

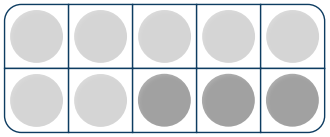
Number pairs within and to 20
Number pairs to 10 are used to support the understanding of number pairs within and to 20.
Introduced in Year 2

Using number pairs to add and subtract
As pupils explore the relationship between number pairs within / to 10 and number pairs within / to 20 (and beyond when they are comfortable with this), they can use this to explore addition and subtraction using number pairs.
Introduced in Year 1

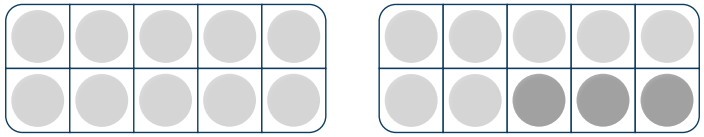
Tens frames

Tens frames can be used to show the link between number pairs that make 10 and number pairs that make 20. These representations show pupils that they can apply their prior knowledge to other calculations.

If we know 7 + 3 = 10

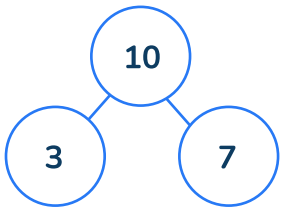


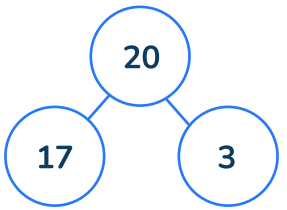
Then we can work out 17 + 3 = 20

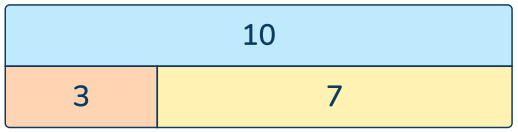


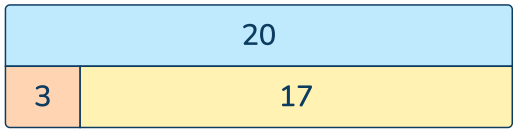
Part-whole model or bar model

When we compare the two part-whole models, we can see that the 10 has increased by 10 in the second model. As one of the parts (3) in the second model is the same, the other part must have increased by 10 to make the new whole.









Number facts

Concept(s)

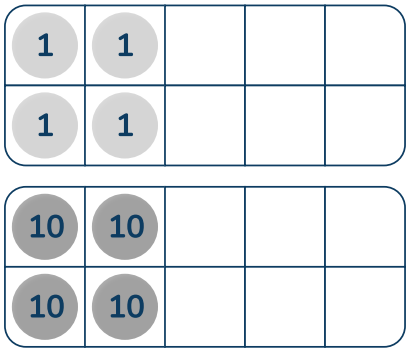
Number pairs to 100
Once pupils are confident with number pairs to 20, they can apply this knowledge to find number pairs to 100.

Initially, pupils use number pairs to 10 to find related number pairs to 100.

Pupils then move on to finding number pairs to 100 using their understanding of partitioning numbers into tens and ones.
Introduced in Year 2

Tens frames and place value counters

Concrete resources can be used to clearly show the relationship between number pairs that make 10 and number pairs to 100.

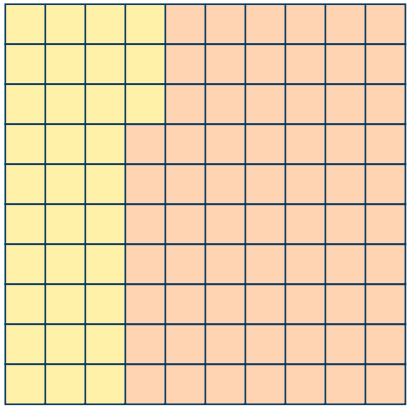


Base 10 drawings

Once pupils are familiar with base 10, they can use drawings to represent base 10. A square represents one hundred, a line represents one ten and a circle represents one one.

Shaded hundred square

Either a blank or numbered hundred square can be used to show number pairs to 100.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Number facts

Concept(s)

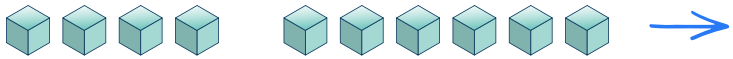
Number pairs to 100
Continued

Pupils can also apply this understanding to number pairs beyond 100, for example number pairs to 1,000
Introduced in Year 3

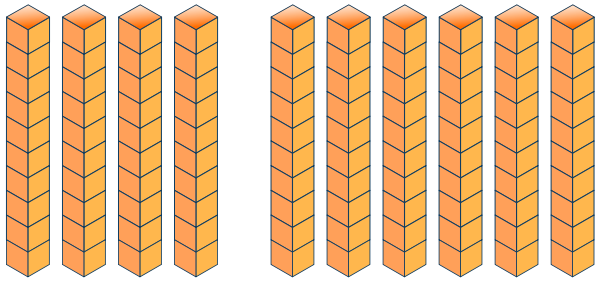
<u>Base 10 / bead strings / Rekenrek</u>

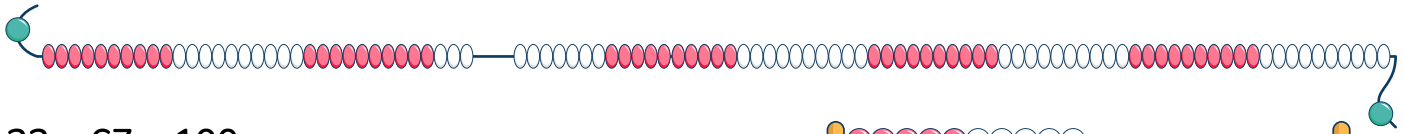
Base 10, bead strings and Rekenreks can all be used to identify number pairs with tens and ones to 100. Bead strings (with 100 beads) and Rekenreks are especially useful as they are grouped into tens / ones with one hundred as the total.

4 + 6 = 10

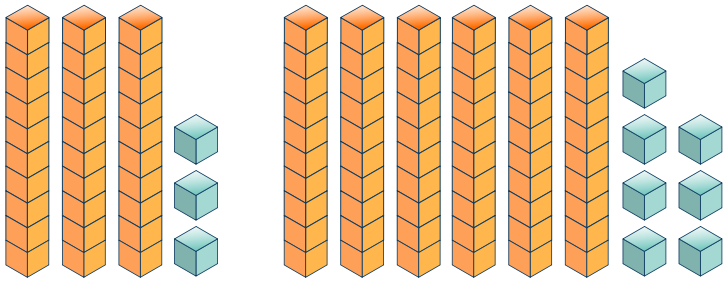


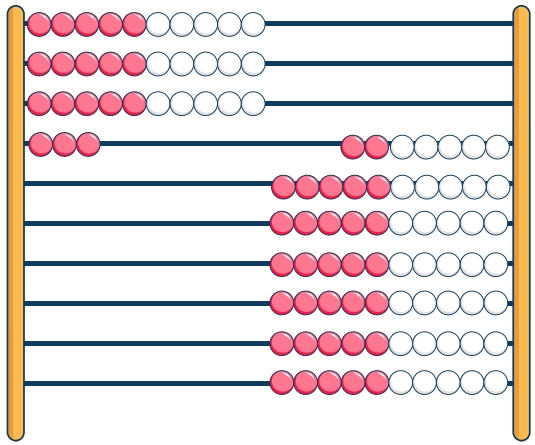
40 + 60 = 100





33 + 67 = 100





Mental and written addition and subtraction

Concept(s)

Count on to add and back to subtract

Pupils are introduced to commutativity.

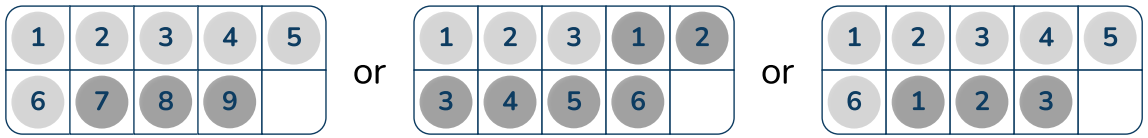
Pupils are taught that they can count on from either addend to find the total.

They also explore counting on from the larger addend to find the total.

Subtracting by counting back is also introduced.

Introduced in Year 1

Counting using tens frames



6 + 3 9 – 6 9 – 3

Counters alone can also be used. Using a tens frame will help pupils identify if their answer is greater / less than a ten (or a multiple of 10).

Hundred square

Hundred squares can be used to identify the larger addend or the minuend and count on / back, rather than counting items individually.

Hundred squares are also useful when exploring finding 1 or 10 more / less than a number.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Mental and written addition and subtraction

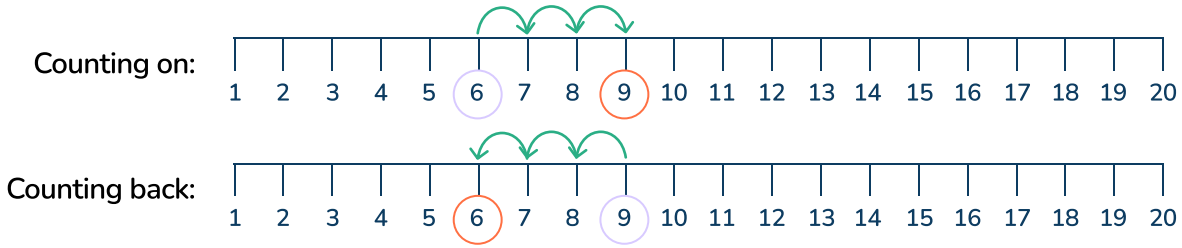
Concept(s)

Count on to add and back to subtract

continued

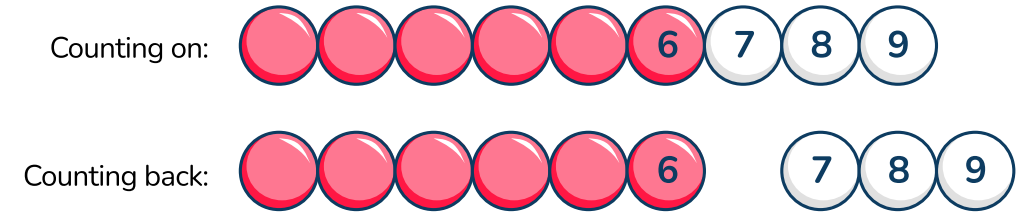
Number lines

Similar to hundred squares, number lines can help pupils work efficiently by counting on from the larger addend or identify the minuend and count back. When pupils work with negative numbers to count on and back, number lines can help pupils understand the numbers they are working with.



Bead string

Bead strings can help pupils understand the concept of counting on or back by physically moving the beads.



Mental and written addition and subtraction

Concept(s)

Counting on to find a difference

Pupils are introduced to the concept of subtracting being the difference.

It is important pupils understand the – symbol means 'difference' AND 'take away' AND 'subtract' AND 'reduce' (and so on), not just one of these.

Introduced in Year 1

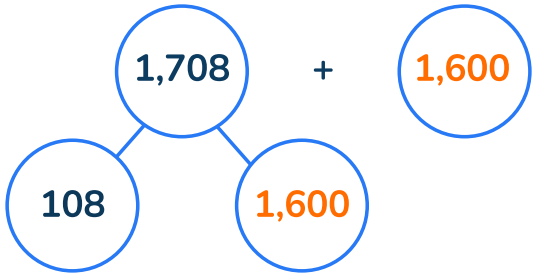
Part-whole model

Part-whole models can be used for larger numbers as well as smaller numbers.

To solve 1,708 + 1,600, we can partition 1,708 into 108 and 1,600.

This will allow us to double 1,600.

We can then add 108.

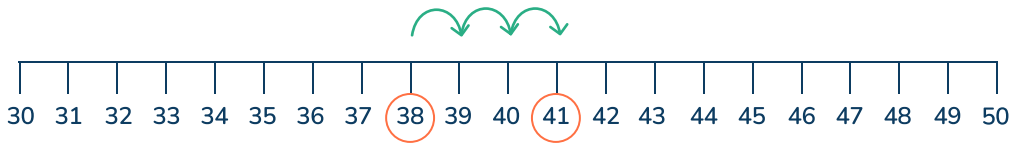


Number line

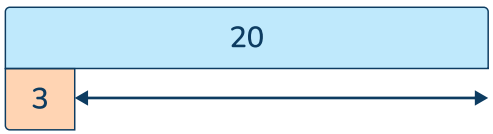
Pupils should be encouraged to identify when counting on is the most efficient strategy.

Number lines can be an excellent way to identify when numbers are close so counting on will be easier than a formal method.

41 – 38 = 3



Bar model as the difference



Abstract

15 – 8 can be read as: What is the difference between 8 and 15?

The same concept can be applied to larger numbers: 1,003 – 997 can be read as: What is the difference between 997 and 1,003?

Mental and written addition and subtraction

Concept(s)

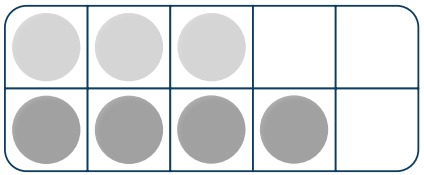
Near doubles

If two numbers are close in value, we can use doubling and adjusting to complete a calculation.

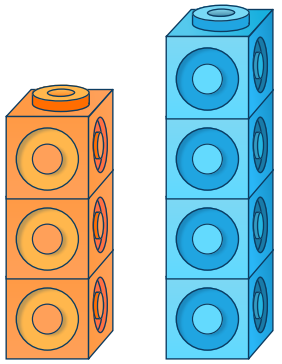
Introduced in Year 1

Lining items up: Counters / tens frames

Lining up counters will help pupils see that the numbers are numerically close. Pupils can then use this information to select the addend they want to double and how to adjust.



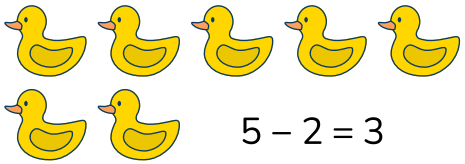
Lining items up: Multi-link (or plain cubes)

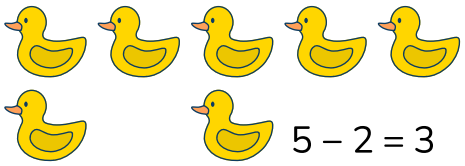


Lining items up: Toys

Lining items up underneath to each other (toys, counters, cubes etc) will help pupils identify the difference between numbers.

Pupils would benefit from having the representations organised like the first image but should also be exposed to other representations such as the second image when they are confident with the meaning of 'difference'.





Lining items up: Number shapes

Mental and written addition and subtraction

Concept(s)

Partitioning

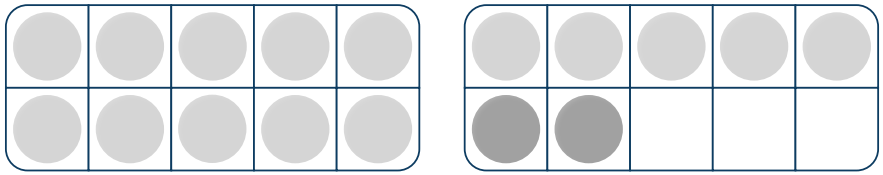
Pupils should use their understanding of tens and ones to initially add the ones, then add the tens. Adding or subtracting the lowest value column first will make it easier for pupils when regrouping or exchanging is involved or when decimals are involved.

Introduced in Year 2

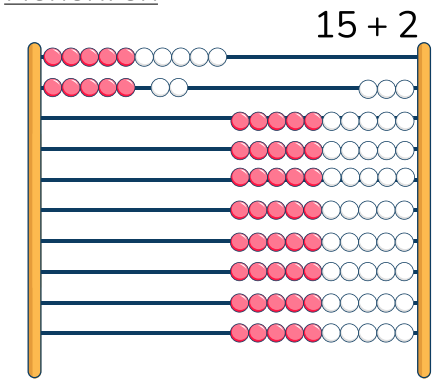
Ten frames and counters

Tens frames and counters work very well with lower value calculations but can become cumbersome with larger numbers.

15 + 2



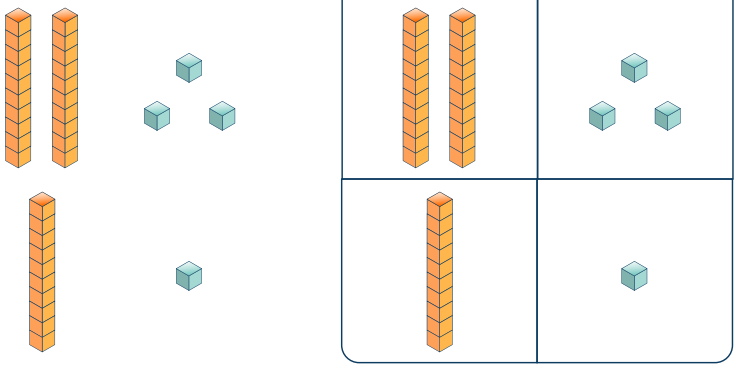
Rekenrek



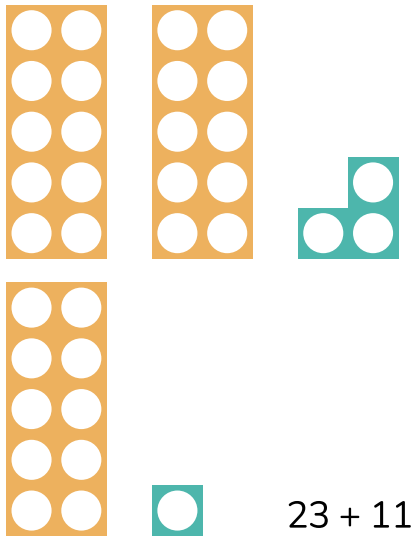
Base 10 without or with a place value chart

For larger numbers, resources that clearly show groups of ten make the process much easier.

23 + 11



Number shapes:



Mental and written addition and subtraction

Concept(s)

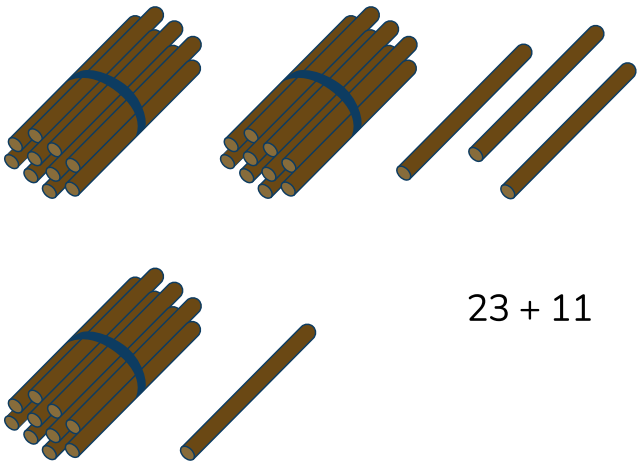
Partitioning
continued

Jottings

Jottings can be used to partition the numbers into tens and ones.

23 + 11 = 34
3 + 1 = 4
20 + 10 = 30
30 + 4 = 34

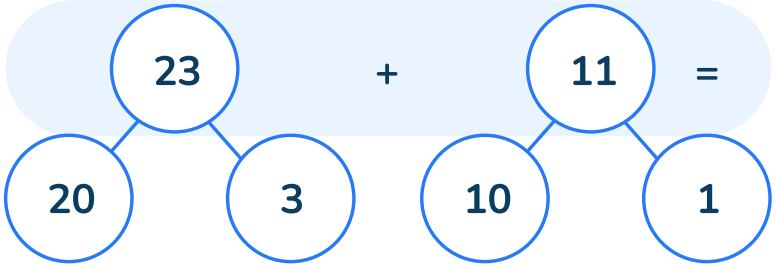
Sticks



23 + 11

Part-whole model

Part-whole models can be used to partition each number.



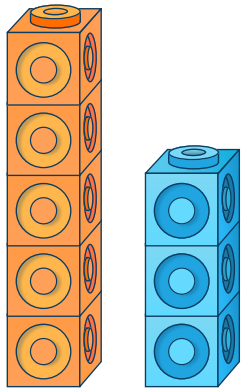
Mental and written addition and subtraction

Concept(s)

Inverse operations
Pupils should understand that there is a relationship between addition and subtraction. Exploring how to use the inverse operation helps pupils develop different checking strategies. This builds upon learning about fact families and applies it to any addition / subtraction calculation.
The term 'inverse operation' is introduced in Year 3

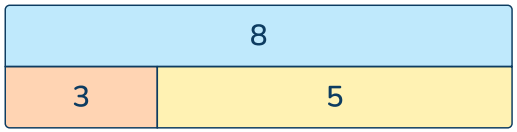
Any concrete resource or representation that clearly shows the whole and parts can be used to help support pupils' understanding of inverse operations.

Multi-link (or plain cubes)

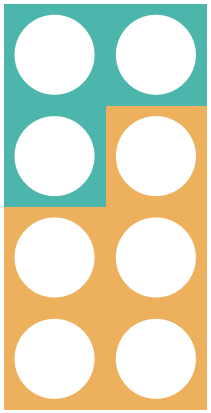


Bar model

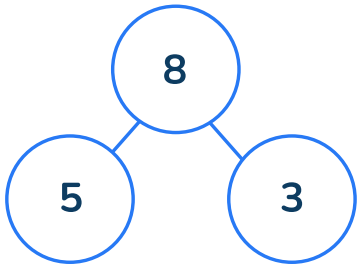
Bar models and part-whole models clearly show the parts and wholes in the calculation.



Number shapes



Part-whole model



Mental and written addition and subtraction

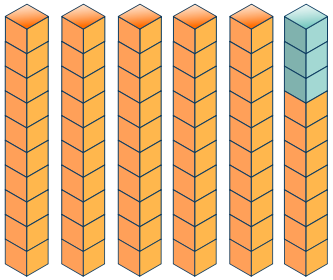
Concept(s)

Rounding and adjusting
Building on the concept of doubling and adjusting, pupils learn to round a number then adjust.
Introduced in Year 3
Note that in Year 3 the language of 'estimate' and 'estimation' are used instead of rounding.

Using resources that clearly show groups of ten can help pupils to identify how to round the numbers they are working with.

125 + 57
125 + 60 = 185
185 – 3 = 182

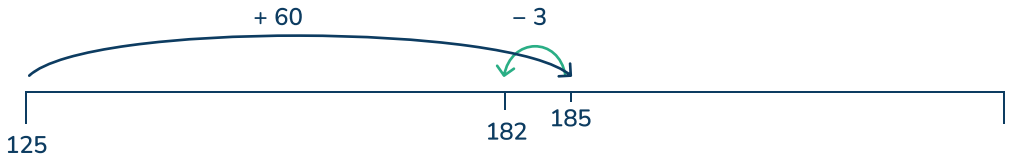
We add the nearest multiple of 10 and then adjust.



We have added 3 too many

<u>Number line</u>

Number lines can be used to clearly show that there are 3 too many so we need to subtract 3.



Mental and written addition and subtraction

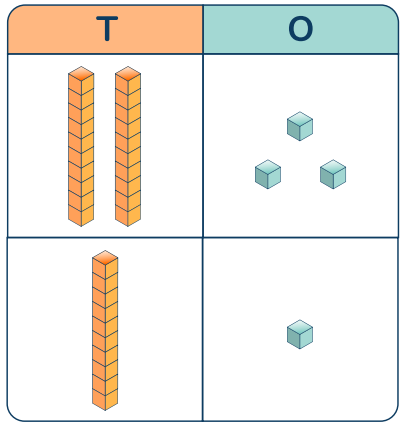
Concept(s)

Formal written methods
Initially, formal written methods are introduced using concrete resources within a place value chart. Generally these use counters or base 10 equipment. Pupils start by adding without regrouping and subtracting without exchanging then move on to adding with regrouping and subtracting with exchanging.
Introduced in Year 3

<u>Place value charts (and Base 10)</u>

Place value charts clearly show the place value of each digit in a calculation. A place value chart can be used to help pupils understand the abstract representation of adding or subtracting numbers using the column method.

23 + 11

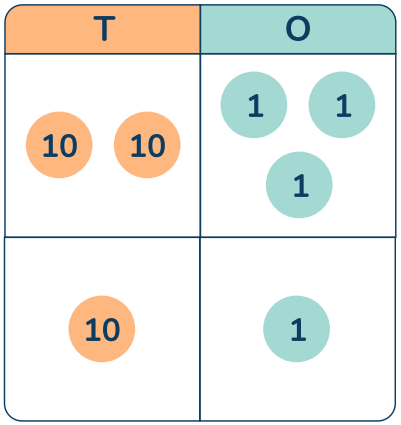


Using concrete resources can help pupils to understand when they need to regroup or exchange within a calculation.

		T	O
		2	9
+		1	7
		4	6
		1	

		T	O
		3 ~~4~~	1 7
–		1	9
		2	8

<u>Place value charts (and counters)</u>



23 + 11

Mental and written addition and subtraction

Concept(s)

Same sum and same difference

This concept builds upon the concept of rounding and adjusting. Pupils should understand that they can adjust a calculation to make it easier to solve mentally.

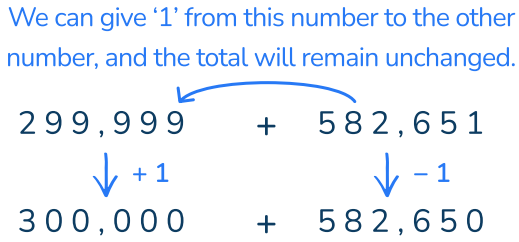
Introduced in Year 5

Once pupils are confident with the same difference method, they can extend this to make simpler subtraction calculations.

Introduced in Year 6

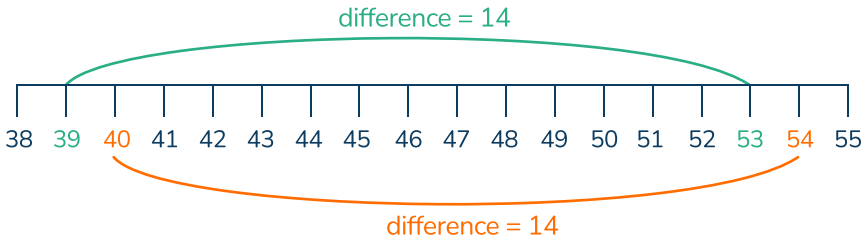
Jottings

Jottings can be used to show how a calculation has been adjusted either using the same sum or same difference.



Number line

A number line can be used to show that the calculation has been adjusted but the sum or difference have stayed the same.



Bar model

Bar models can be used to clearly illustrate why the same difference method works, especially when making a calculation simpler. While a multiple of 10 / 100 / 1,000 etc will seem simpler, if pupils solve this calculation using a formal written method, there is a lot of room for error.

16,000 – 4,469 = 15,999 – 4,468