Multiplication and Division

In Multiplication and division, we have 3 key teaching concepts:

1. Mental / informal methods before written methods when solving problems.
2. Understanding the relationship between multiplication and division.
3. Correct terminology: regrouping (multiplication) and exchanging (division)

1. Mental / informal methods before written methods when solving problems.

When solving multiplication or division calculations, it is important for pupils to consider whether they can solve it in their heads with jottings (mentally) rather than using a formal written method. There are a few different strategies pupils may choose to use. Encouraging pupils to talk through the method they are using (or are considering using) when approaching a problem. This will help develop their mathematical language and reasoning skills.
It is important when teaching and modelling the formal method for multiplication and division that the correct language is used and we focus on the value of the digits throughout. As with addition and subtraction, pupils should not think they are only ever working with ones.

2. Understanding the relationship between multiplication and division.

It is important that pupils don't see multiplication and division as two separate things. Instead we want to draw attention to the relationship between them. We can help pupils to see the connections by using arrays, fact triangles or diagrams / scenarios and these are used throughout the sessions. To reinforce the knowledge of the relationship between multiplication and division, you can encourage pupils to check division calculations by using multiplication and vice versa. Use of this strategy becomes particularly useful when solving missing number problems and will help students to solve more complex calculations, such as 'Working backwards' problems.

3. Correct terminology: regrouping (multiplication) and exchanging (division)

As with addition and subtraction, it is useful to use consistent language with multiplication and division. At Third Space Learning, we use ‘regrouping’ in multiplication and ‘exchanging’ in division. This will also reinforce the concept of multiplication as repeated addition and division as repeated subtraction.
It is important when teaching and modelling the formal method for division that the correct language is used and we focus on the value of the digits throughout. Division is the only operation where we start with the most significant digit first (ie. the highest value digit.)

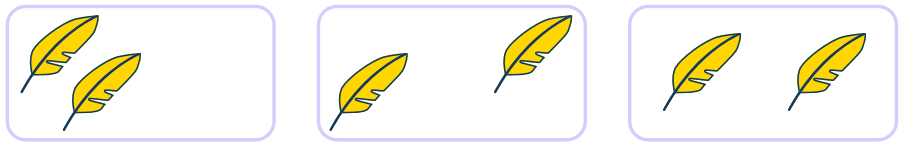
Multiplication and division facts

Concept(s)

Multiplication as repeated addition (equal groups)
Multiplication is initially taught through recognising and adding equal groups.
E.g. 2 + 2 + 2 = 6
Introduced in Year 1

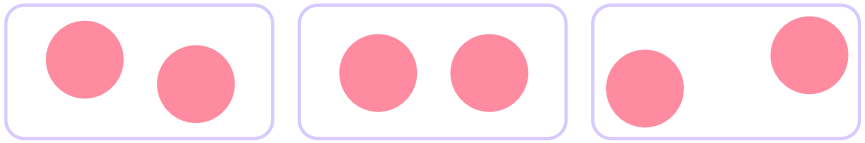
When pupils are familiar with adding equal groups, they can relate this to multiplication calculations.
E.g. 2 + 2 + 2 = 6
3 lots of 2 = 6
3 × 2 = 6
Introduced in Year 2

<u>Toys or everyday objects</u>
Pupils should use familiar resources to make and count equal groups. They should be encouraged to understand that the groups do not need to look identical, as long as each group contains the same number of items.

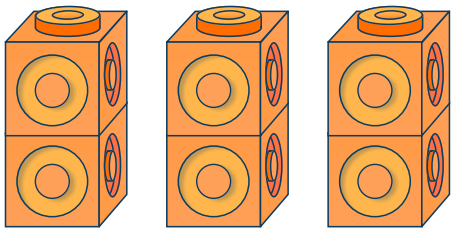


Maths manipulatives can then be used instead of toys or everyday objects.

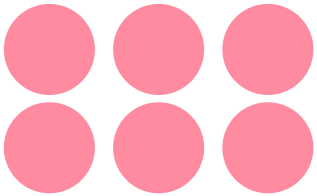
<u>Counters</u>



<u>Multi-link (or plain cubes)</u>



<u>Arrays</u>
Arrays can be introduced to begin to explore the commutative nature of multiplication.



Multiplication and division facts

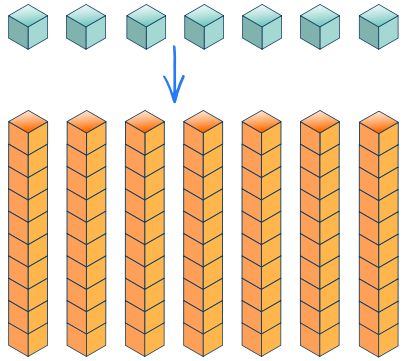
Concept(s)

Multiplication as scaling
Pupils are taught that multiplication means [number] times the size.

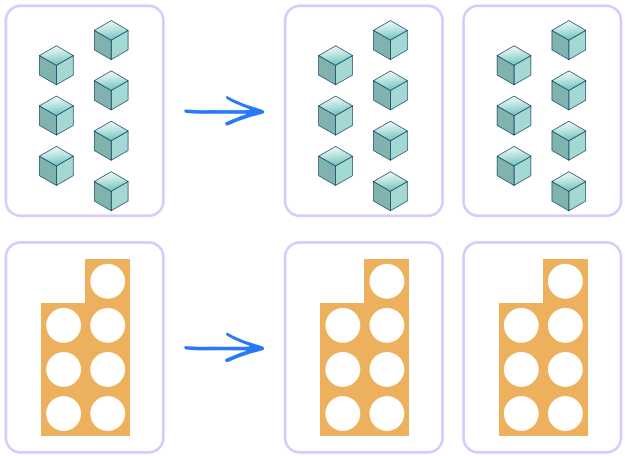
Initially, pupils look at doubling and ten times to understand scaling.
Concept introduced in Year 1
Term scaling introduced in Year 3

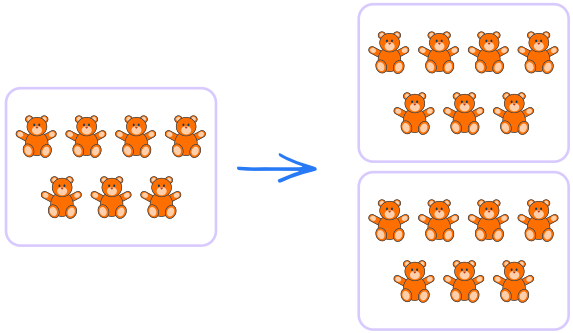
<u>Base 10 and place value counters</u>
Multiplying by 10 (7 × 10)

Pupils can use Base 10 or place value counters to show that we are making a number (in this case 7) ten times the size.



<u>Base 10, number shapes, toys / everyday objects</u>
Doubling (7 × 2)





Pupils can use various resources (including toys and everyday objects) to show that the original number is twice the size.

Multiplication and division facts

Concept(s)

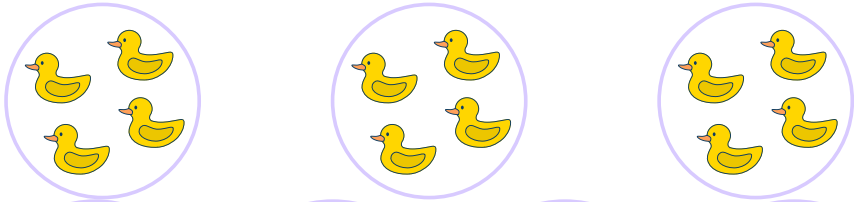
Division as sharing and grouping

Pupils look at how division can either be thought of as sharing or grouping. This also reinforces that division involves equal groups.

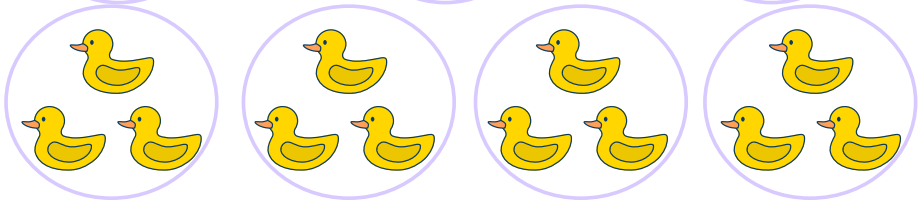
Sharing and grouping are introduced in Year 1. Division with the division symbol is introduced in Year 2.

Using the same resources, pupils can either group or share to divide.

Sharing between 3 groups



Grouping into 3s

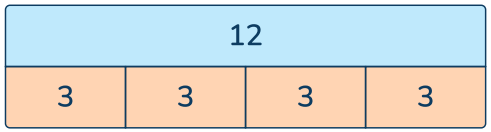


Toys, counters, cubes, multilink, can all be used to model and explore between sharing and grouping.

<u>Simple pictorial representations</u>

<u>Bar models</u>

The bar model shows there are 4 groups of 3 or that 12 has been shared equally between 4 groups.



Multiplication and division facts

Concept(s)

Developing multiplication and division facts (up to 12 x 12)

In our intervention lessons, we focus on teaching different methods to learn the multiplication tables. These methods involve finding patterns in a hundred square, skip counting and identifying / using relationships.

Hundred square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5 times table

10 times table

All numbers that are in the 2x table are even.

If halving a number gives an even value, then the number is in the 4 x table.

If halving a number twice gives an even value, then the number is in the 8 x table.

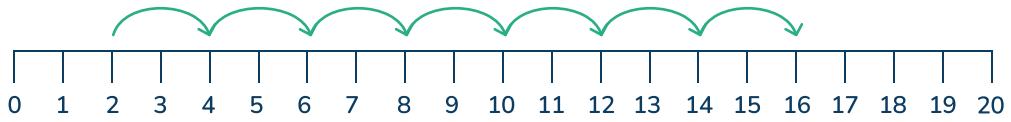
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2 times table

8 times table

4 times table

Number lines (skip counting)



Pupils can use the number line to count in multiples of a given number.

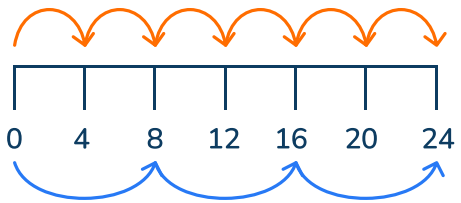
Multiplication and division facts

Concept(s)

Developing multiplication and division facts (up to 12 x 12)
continued

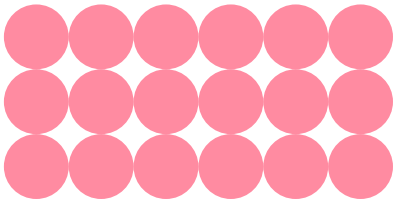
Identifying and using relationships

6 × 4 = 24
3 × 8 = 24



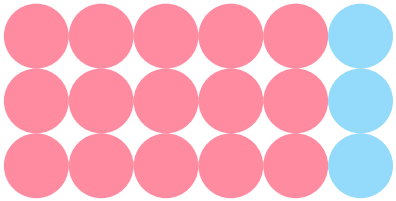
Arrays

Arrays can be used to show a fact family. Pupils can use this to identify all the calculations related to the given array. Arrays can be created using concrete resources or represented pictorially.



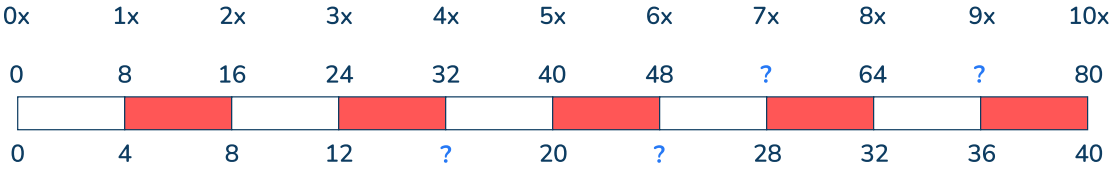
$6 \times 3 = 18,\ 3 \times 6 = 18,\ 18 \div 6 = 3,\ 18 \div 3 = 6$

Arrays can also be used to highlight known facts (the distributive law)



I know $5 \times 3 = 15$ so $6 \times 3 = 5 \times 3 + 3 = 18$

Pupils can also use number lines to identify and use relationships between multiplication tables. This example shows using doubling of the 4 times tables to solve the 8 times tables. This method can be applied to a counting stick.



Informal methods

Concept(s)

Partitioning using an area model to multiply and divide
This is where we split one of the numbers into two parts and then multiply and / or divide each part.
Introduced in Year 3

Initially, pupils multiply or divide by 1-digit numbers then they move on to 2-digit numbers.
Introduced in Year 4

Arrays

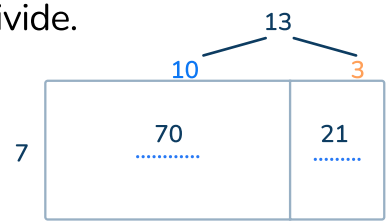
Arrays can be used to partition a number into more manageable parts to multiply or divide.

13 x 4

Area model

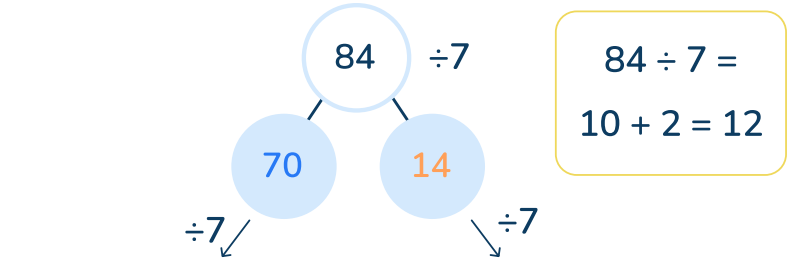
An area model builds on partitioning numbers to multiply and divide.

7 x 13



Part-whole models

When pupils are confident with partitioning to multiply and divide, they can use the pictorial representation of a part-whole model to partition a number then multiply or divide.



Distributive Law

Pupils should understand that the distributive law means distributing the multiplication operation over addition or subtraction and multiplying by each part of the number. 7 x 13 = (7 x 10) + (7 x 3)

Informal methods

Concept(s)

Using factors to multiply and divide

Once pupils have a secure understanding of factors, they can use factors to break one number then multiply or divide. Pupils need to have a secure understanding of factors to avoid confusion with partitioning and finding factors.

Introduced in Year 5

Associative law

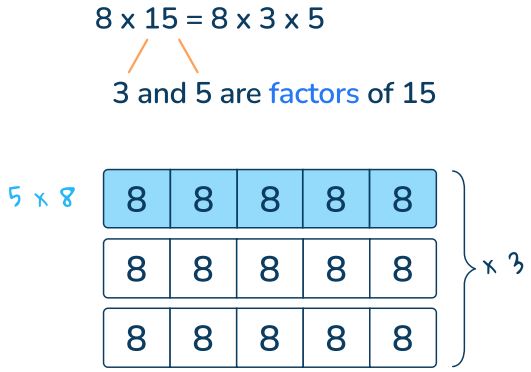
Pupils should understand that the order they multiply does not matter.

8 x 3 x 5 = 5 x 8 x 3

Arrays / bar models

Arrays can be used to represent the factors of one number.

The arrays can be created using concrete resources (such as counters) or bar models, although the bar models are slightly clearer for larger numbers than an array would be.



Jottings

Jottings can be used to partition one number and solve the calculation in steps.

90 ÷ 6

2 × 3

2 and 3 are factor of 6

First, divide 90 by 2 or 3.

90 ÷ 3 = 30

Next, divide the answer by the other factor

30 ÷ 2 = 15

Therefore 90 ÷ 6 = 15

Informal methods

Concept(s)

Using known facts and adjusting

There are some multiplication table facts that pupils find easier to calculate with than others (e.g. 2s, 5s, 10s). Pupils can use known facts to help them solve calculations quickly.

Introduced in Year 4

<u>Jottings</u>

Pupils can use jottings to identify the known facts and the adjustment needed to solve the original calculation.

To solve 31 x 8, we can use known facts:

31 x 10 = 310

This is 2 lots of 31 too many

310 – (2 x 31) =

310 – 62 = 248

So

31 x 8 = 248

Formal Written Methods

Concept(s)

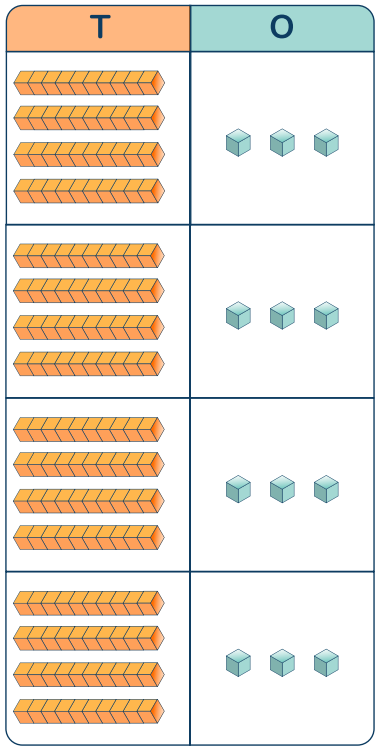
Expanded method

The expanded method of multiplication encourages the understanding of place value when multiplying.

Introduced in Year 4 for 1-digit x 2 or more digits

Place value chart and Base 10 or counters

Place value charts can be used with mathematical manipulatives to reinforce the understanding of place value when first introducing formal written multiplication.



Formal written method

		T	O	
		4	3	
x			4	
		1	2	(4 x 3)
+	1	6	0	(4 x 40)
	1	7	2	

		2	3	1	
x			3	4	
				4	(4 x 1)
		1	2	0	(4 x 30)
		8	0	0	(4 x 200)
			3	0	(30 x 1)
		9	0	0	(30 x 30)
+	6	0	0	0	(30 x 200)
	7	8	5	4	
	1				

Formal Written Methods

Concept(s)

Short multiplication
Short multiplication is used when multiplying large numbers by a single digit number.
Pupils need to remember to multiply each digit in turn.
Introduced in Year 4

Formal written method

		T	O
		4	3
x			4
	1	7	2
	1	1	

	H	T	O
	3	5	2
x			4
1	4	0	8
1	2		

Steps for 43 x 4
1) 4 x 3 ones = 12 ones (we can regroup 10 ones for 1 ten)
2) 4 x 4 tens = 16 tens
16 tens + 1 ten = 17 tens

Formal Written Methods

Concept(s)

Long multiplication
Long multiplication is used when multiplying by larger numbers.
As with short multiplication, it is important pupils understand the place value of each digit in the calculation. This ensures pupils understand the reason for their answers. For example, putting a place holder on the second line of the calculation shows multiplying in the tens.
Introduced in Year 5

Formal written method

		3	1	2
x			2	6
	1	8	7 (1)	2
+	6	2	4	0
	8	1	1	2
	1	1		

Steps to solve 312 x 26
Start with the ones in the multiplicand (26).
1) 6 x 2 ones = 12 ones (we can regroup 10 ones for 1 ten)
2) 6 x 1 ten = 6 tens
6 tens + 1 ten = 7 tens
3) 6 x 3 hundreds = 18 hundreds

Then, multiply by the tens in the multiplicand (26)
4) 2 (tens) x 2 ones = 4 tens
5) 2 (tens) x 1 ten = 2 hundreds
6) 2 (tens) x 3 hundreds = 6 thousands

Formal Written Methods

Concept(s)

Short division (with and without exchanging)

Short division is used when dividing by a single digit number or a number that a pupil knows its multiplication tables for.

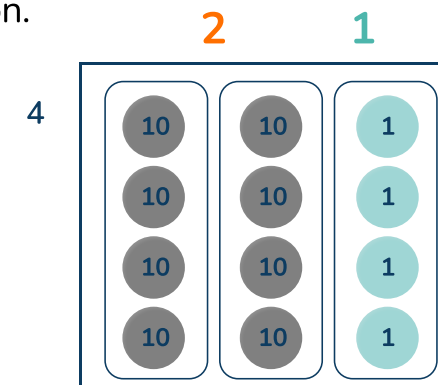
Pupils start with the largest value digit and should use the correct place value language. For example, we are not dividing 6, we are dividing 6 hundreds.

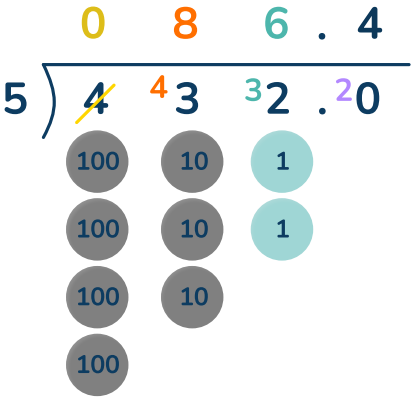
Introduced in Year 5

Place value counters or Base 10

Place value counters can be used to help pupils identify the place value of the digits in the calculation.

84 ÷ 4 = 21





Formal written method

Steps to solve 687 ÷ 5

1) Divide the hundreds:

6 hundreds ÷ 5 = 1 hundred r 1 hundred

2) Exchange the hundreds for tens: 1 hundred = 10 tens

10 tens + 8 tens = 18 tens

3) Divide the tens: 18 tens ÷ 5 = 3 tens r 3 tens

4) Exchange the tens for ones: 3 tens = 30 ones

30 ones + 7 ones = 37 ones

5) Divide the ones: 37 ones ÷ 5 = 7 ones r 2 ones

	H	T	O	
	1	3	7	r. 2
5	6	1 8	3 7	

Formal Written Methods

Concept(s)

Long Division

Long division is used when dividing by a 2-digit number. Pupils start with the largest value digit and pupils may need to exchange.

It is important to maintain the value of the digits throughout. For example, we are not dividing 4, we are dividing 4 hundreds.

Introduced in Year 6

Formal written method

Place value counters or base 10 can again be used to ensure pupils understand the value of the digits they are working with. Listing multiples of the divisor in a fact box can help with division.

Steps to solve 434 ÷ 31

1) Divide the hundreds:

4 hundreds ÷ 31 = 0 hundred

2) Exchange the hundreds for tens:

4 hundred = 40 tens

40 tens + 3 tens = 43 tens

3) Divide the tens:

43 tens ÷ 31 = 1 ten r 12 tens

4) Exchange the tens for ones:

12 tens = 120 ones

120 ones + 4 ones = 124 ones

5) Divide the ones:

124 ones ÷ 31 = 4 ones

(We can use a list of multiples of 31 to find this fact.)